GASP: Gradient Aware Shortest Path Algorithm for Boundary-Confined Visualization of 3D Reeb Graphs

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Introduction

What is a Reeb graph?

A Reeb graph is a mathematical concept used to analyze and understand the topology (shape and structure) of a manifold or scalar field, particularly in multidimensional data.

Why are Reeb graphs important?

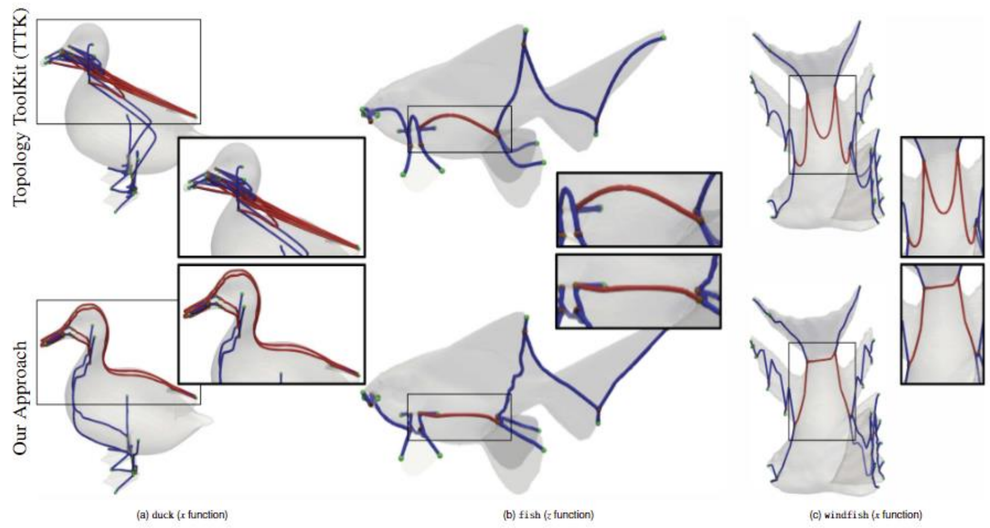
Reeb graphs play a crucial role in the structural analysis and visualization of scalar field data on manifolds, applicable in diverse fields from medical imaging to geography.

Motivation

Despite their utility, existing visualization techniques often misrepresent the data through arcs that extend beyond the model boundary, follow unnecessarily long paths, or inadequately align with the data's gradient flow. The paper introduces a new algorithm, GASP, aimed at improving the visualization of 3D Reeb graphs. By focusing on boundary-constrained, shortest-path, and gradient-aware visualizations, GASP addresses shortcomings in existing methodologies like the Topology ToolKit (TTK), particularly in accurately representing the underlying height function of models.

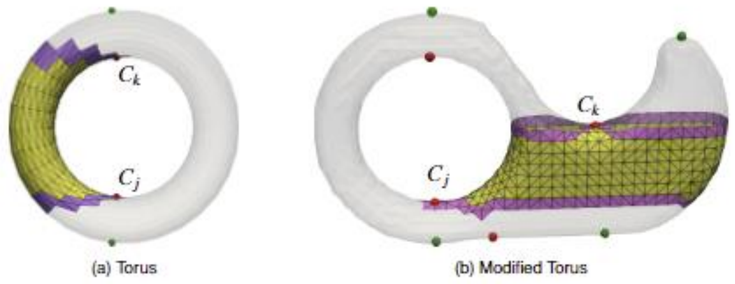
Challenges Addressed:

- **Arcs Outside the Model:** Traditional approaches may generate Reeb graph arcs that extend beyond the model's boundary, leading to inaccurate representations.
- **Unnecessarily Long Arcs:** Existing methodologies can result in Reeb graph edges that are longer than needed due to smoothing processes that disregard the shortest possible path.
- **Misalignment with Gradient:** Previous methods sometimes fail to align Reeb graph arcs with the function's gradient, compromising the visualization's fidelity to the actual data structure.

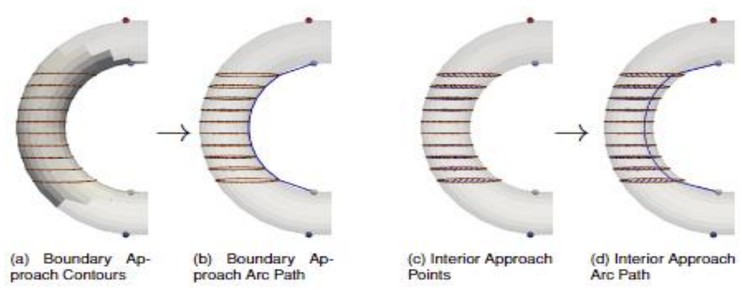


Proposed Solution: GASP Algorithm

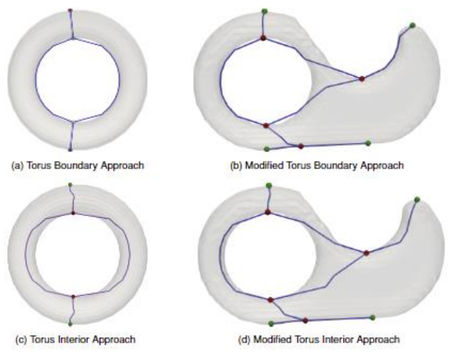
❖ **Step 1 Decomposition**: The model is decomposed into topological cylinders associated with Reeb graph edges.



❖ **Step 2 Reeb Graph Arc Construction:** For each cylinder, GASP calculates the shortest path that conforms to the model's boundary and aligns with the gradient of the height function.



❖ **Step 3 Final Assembly:** The individual arcs are then assembled to form the final Reeb graph visualization.

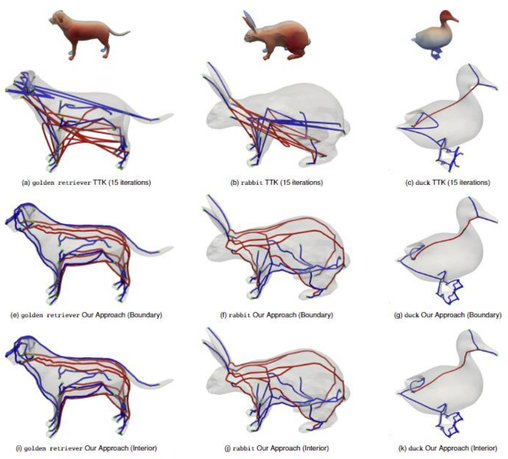


This approach ensures that Reeb graph arcs remain within or on the surface of the object, take the shortest route between critical points while being constrained by the boundary, and better align with the elevation function's gradient.

Evaluation

Boundary Constraint

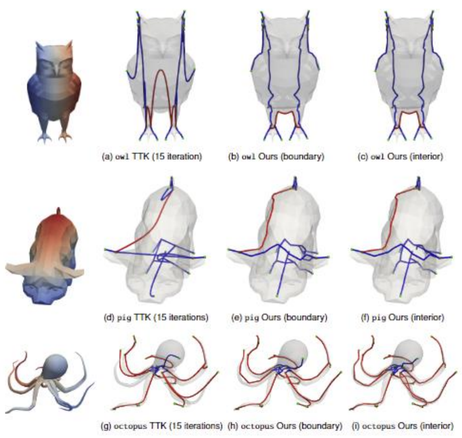
Measuring how much of the Reeb graph exists of the model



dataset	method	x-function # arcs	x-function avg	x-function max	y-function # arcs	y-function avg	y-function max	z-function # arcs	z-function avg	z-function max
bird	Ours/Boundary	53	**0.009**	0.475	47	0.014	0.400	25	**0.000**	**0.000**
	Ours/Interior		**0.009**	0.475		0.001	0.061		**0.000**	**0.000**
	TTK/5		0.015	**0.347**		**0.000**	**0.000**		0.032	0.244
	TTK/15		0.033	0.667		0.018	0.379		0.042	0.360
duck	Ours/Boundary	73	0.014	0.514	47	0.038	1.000	45	0.019	0.391
	Ours/Interior		**0.005**	**0.235**		0.009	0.236		**0.000**	**0.000**
	TTK/5		0.139	1.000		**0.007**	0.232		0.038	0.984
	TTK/15		0.144	1.000		**0.007**	**0.226**		0.051	0.982
fish	Ours/Boundary	21	0.095	1.000	33	0.017	0.482	27	0.037	0.767
	Ours/Interior		**0.000**	**0.000**		0.002	0.076		0.040	1.000
	TTK/5		**0.000**	**0.000**		**0.000**	**0.000**		0.008	0.224
	TTK/15		**0.000**	**0.000**		0.014	0.153		**0.004**	**0.121**
foot1	Ours/Boundary	29	**0.000**	**0.000**	17	0.008	0.132	21	0.016	0.201
	Ours/Interior		**0.000**	**0.000**		**0.000**	**0.000**		**0.000**	**0.000**
	TTK/5		0.014	0.398		**0.000**	**0.000**		**0.000**	**0.000**
	TTK/15		0.021	0.446		0.019	0.327		**0.000**	**0.000**
golden retriever	Ours/Boundary	71	**0.017**	**0.376**	49	0.030	1.000	73	0.026	1.000
	Ours/Interior		**0.017**	**0.376**		0.006	0.304		**0.010**	**0.287**
	TTK/5		0.127	1.000		**0.003**	**0.148**		0.048	0.960
	TTK/15		0.108	1.000		0.008	0.221		0.064	1.000
heart	Ours/Boundary	33	0.017	0.556	9	0.063	0.320	33	0.025	0.825
	Ours/Interior		**0.000**	**0.000**		**0.000**	**0.000**		**0.000**	**0.000**
	TTK/5		**0.000**	**0.000**		**0.000**	**0.000**		0.047	1.000
	TTK/15		**0.000**	**0.000**		**0.000**	**0.000**		0.039	0.412
octopus	Ours/Boundary	58	0.063	1.000	56	0.072	0.747	40	0.036	0.444
	Ours/Interior		**0.000**	**0.000**		**0.016**	**0.495**		**0.002**	**0.085**
	TTK/5		0.158	1.000		0.098	0.673		0.135	1.000
	TTK/15		0.248	1.000		0.174	0.946		0.254	1.000
ok hand	Ours/Boundary	34	**0.019**	**0.230**	28	0.075	0.643	50	0.023	0.885
	Ours/Interior		**0.019**	**0.230**		**0.000**	**0.000**		**0.000**	**0.000**
	TTK/5		0.079	0.676		**0.000**	**0.000**		0.076	1.000
	TTK/15		0.097	1.000		0.002	0.068		0.066	1.000

Shortest Path Arcs

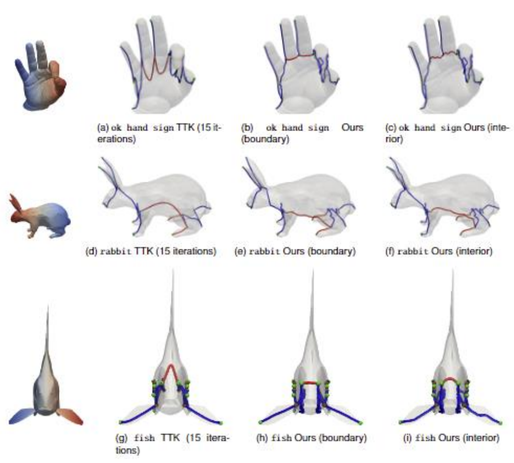
Measuring the total Reeb graph length



dataset	method	x-function # arcs	x-function avg	x-function max	y-function # arcs	y-function avg	y-function max	z-function # arcs	z-function avg	z-function max
bird	Ours/Boundary	53	**1.07**	**1.72**	47	1.14	2.08	25	1.09	1.40
	Ours/Interior		1.14	4.74		1.14	2.08		1.11	1.40
	TTK/5		1.80	14.36		1.05	1.35		1.08	1.65
	TTK/15		1.64	12.97		**1.02**	**1.10**		**1.04**	**1.22**
duck	Ours/Boundary	73	**1.13**	**4.61**	47	**1.08**	**1.38**	45	1.20	2.90
	Ours/Interior		1.13	**4.61**		1.09	**1.38**		1.24	2.90
	TTK/5		3.22	18.92		1.29	2.75		1.12	2.05
	TTK/15		2.40	13.31		1.12	2.36		**1.07**	**1.56**
fish	Ours/Boundary	21	**1.07**	**1.39**	33	**1.15**	**3.37**	27	1.10	1.91
	Ours/Interior		1.13	2.02		1.19	**3.37**		1.17	1.91
	TTK/5		1.66	10.54		1.26	6.42		1.13	1.83
	TTK/15		1.46	8.70		1.19	6.07		**1.09**	**1.67**
foot1	Ours/Boundary	29	**1.32**	**6.05**	17	**1.08**	**1.25**	21	1.13	**1.66**
	Ours/Interior		1.37	**6.05**		1.09	**1.25**		1.13	**1.66**
	TTK/5		3.20	45.42		1.48	3.05		1.15	2.61
	TTK/15		2.75	37.57		1.33	2.49		**1.10**	2.42
golden retriever	Ours/Boundary	71	**1.08**	1.64	49	**1.11**	**2.50**	73	1.19	4.40
	Ours/Interior		1.11	**1.62**		1.13	**2.50**		1.22	4.40
	TTK/5		1.93	30.57		1.19	3.92		1.23	2.39
	TTK/15		1.62	28.22		1.12	3.15		**1.11**	**1.85**
heart	Ours/Boundary	33	**1.06**	**1.50**	9	**1.08**	1.29	33	**1.10**	**1.91**
	Ours/Interior		1.09	**1.50**		1.12	**1.26**		1.12	**1.91**
	TTK/5		1.79	6.17		1.29	1.75		2.08	16.73
	TTK/15		1.61	5.71		1.17	1.62		1.76	13.04
octopus	Ours/Boundary	58	**1.19**	**2.27**	56	1.25	**2.43**	40	1.34	3.28
	Ours/Interior		1.20	**2.27**		1.27	**2.43**		1.37	3.28
	TTK/5		1.46	5.96		1.35	4.34		1.30	2.78
	TTK/15		1.25	4.89		**1.19**	2.99		**1.18**	**2.35**
ok hand	Ours/Boundary	34	**1.12**	**2.54**	28	1.07	1.45	50	**1.08**	**1.56**
	Ours/Interior		1.13	**2.54**		**1.05**	**1.36**		1.08	**1.56**
	TTK/5		1.57	5.23		1.16	2.93		1.72	13.75
	TTK/15		1.32	4.27		1.12	2.61		1.37	8.41

Gradient Aware

Measuring how well each edge direction matches the function gradient



dataset	method	x-function # arcs	x-function avg	x-function max	y-function # arcs	y-function avg	y-function max	z-function # arcs	z-function avg	z-function max
bird	Ours/Boundary	53	**35.2**	566.8	47	8.4	**32.9**	25	1.8	**14.1**
	Ours/Interior		35.3	566.8		**8.4**	**32.9**		1.9	**14.1**
	TTK/5		59.7	**566.7**		19.4	126.7		1.7	**14.1**
	TTK/15		55.6	**566.7**		18.2	105.3		**1.6**	**14.1**
duck	Ours/Boundary	73	**15.1**	**230.2**	47	**11.7**	**63.7**	45	**3.8**	**22.5**
	Ours/Interior		15.1	**230.2**		11.7	**63.7**		3.8	**22.5**
	TTK/5		55.1	780.9		16.0	75.0		4.7	**22.5**
	TTK/15		42.7	540.5		14.0	75.0		4.5	**22.5**
fish	Ours/Boundary	21	**22.6**	**104.9**	33	**5.4**	**36.0**	27	5.8	79.0
	Ours/Interior		22.7	**104.9**		5.5	**36.0**		5.9	79.0
	TTK/5		48.9	277.8		10.9	81.4		4.9	**79.0**
	TTK/15		45.9	228.2		10.4	74.8		**4.8**	**79.0**
foot1	Ours/Boundary	29	**5.9**	**53.8**	17	5.3	**18.0**	21	4.0	**9.4**
	Ours/Interior		6.0	**53.8**		**5.3**	**18.0**		**4.0**	**9.4**
	TTK/5		40.6	408.1		10.6	48.0		5.2	23.5
	TTK/15		34.6	318.1		9.2	46.6		4.5	20.2
golden retriever	Ours/Boundary	71	**11.1**	**134.2**	49	**50.0**	**523.1**	73	**12.3**	**175.5**
	Ours/Interior		11.2	**134.2**		50.0	**523.1**		12.3	**175.5**
	TTK/5		61.0	1313.9		178.4	5986.5		24.6	431.2
	TTK/15		51.9	1212.4		177.9	5986.5		21.1	431.2
heart	Ours/Boundary	33	**6.1**	**31.9**	9	**5.5**	**31.4**	33	**17.9**	**205.1**
	Ours/Interior		6.1	**31.9**		5.6	**31.4**		17.9	**205.1**
	TTK/5		12.4	95.7		6.0	31.4		38.1	245.0
	TTK/15		10.8	95.7		5.7	31.4		36.0	245.0
octopus	Ours/Boundary	58	**21.2**	**911.8**	56	**5.6**	**73.6**	40	**22.2**	778.5
	Ours/Interior		21.2	**911.8**		5.6	**73.6**		22.2	778.5
	TTK/5		65.3	1172.0		161.6	5332.0		24.0	**778.3**
	TTK/15		64.2	1172.0		160.7	5332.0		23.4	**778.3**
ok hand	Ours/Boundary	34	**8.8**	41.8	28	**7.8**	**34.6**	50	**16.2**	**259.8**
	Ours/Interior		8.8	41.8		7.9	**34.6**		16.2	**259.8**
	TTK/5		12.8	57.6		13.4	77.5		61.9	1470.7
	TTK/15		11.6	**41.8**		13.4	77.5		58.5	1470.7

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